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Invariant Connections of Higher Order on Homogeneous Spaces

Koneksje niezmiennicze wyższych rzędów na przestrzeniach jednorodnych

Инвариантные связности высших порядков на однородных многообразиях

The paper contains a construction of higher order connections on a given principal fibre bundle over a homogeneous differentiable manifold. We work with an Ehresmann groupoid which is associated with this bundle. We consider its r-th prolongation and we construct a certain connection of order q. We prove that the obtained connection is invariant with respect to the group action convenably prolonged. For properties of this connection cf. [7].

I. PRELIMINARIES AND NOTATIONS

Let (H,B,G,π) be a principal fibre bundle over a manifold B. Denote by Φ the groupoid which is associated with

(H,B,G,π). Thus elements of Φ are G-isomorphisms of fibres over B. Thus Φ is a smooth manifold which is provided with the two projections, a and b, viz. if $\theta \in \Phi$ sends a fibre through $\pi^{-1}(m)$ to $\pi^{-1}(m')$ then we set $a\theta = m$ and $b\theta = m'$. It is easy to see that if we are given any two points h and k then there exists exactly one element $\theta_{h,k} \in \Phi$ such that θ_{hk} sends the fibre through h to the fibre through k, in such a way that for each $g \in G$ it holds $\theta_{hg,kg} = \theta_{h,k}$. The element which is reciprocal to a given $\theta \in \Phi$ will be denoted by $\sigma\theta$. Evidently we have $\sigma\theta_{h,k} = \theta_{kh}$ and $a\sigma\theta = b\theta$. We define a mapping $\psi : \Phi \times H \longrightarrow H$ by $\psi(\theta,h) = \theta(h)$. If $x \in B$ then we denote the identity mapping of $\pi^{-1}(x)$ by $\tilde{x}$.

If we fix any $m_0 \in B$ then $\{\theta \in \Phi \mid a\theta = m_0\}$ is a fibre bundle which is isomorphic with (H,B,G,π). Analogously $\{\theta \in \Phi \mid b\theta = m_0\}$ is some bundle which is called a co-bundle of (H,B,G,π).

Thus $\{\theta \in \Phi \mid a\theta = b\theta = m_0\}$ is a group and it is isomorphic to G.

We shall use standard notations of jet calculus [1 - 5], but if necessary we introduce and explain some new ones. Thus α and β denote, respectively, the source and the target projections.

$\tilde{J}^r(B,\Phi)$ denotes the set of non-holonomic jets of order r from the manifold B to the manifold Φ. Thus $\tilde{J}^r(B,\Phi)$ has a natural structure of a groupoid over $\tilde{J}^r(B,B)$, [2]. a^r and b^r are the prolonged mappings a and, respectively, b. It maps $\tilde{J}^r(B,\Phi)$ onto $\tilde{J}^r(B,B)$. If we fix some m then $\{X \in \tilde{J}^r(B,\Phi) \mid \alpha(a^r(X)) = m\}$ is a principal fibre bundle over

B. [9]. x being any point of B, ρ_x denotes the mapping which sends all points of B to x and we put $\rho_x^r := j_x^r \rho_x$. Then we introduce the space $\tilde{Q}^r(x)$ to be $\tilde{J}^r(B,\Phi)$ restricted to

$$\{X \mid \alpha(X) = x, \quad \beta(X) = \tilde{x}, \quad a^r(X) = \rho_x^r, \; b^r(X) = j_x^r 1_B\}.$$

$\tilde{Q}^r(x)$ is a fibre over x of a certain bundle $\tilde{Q}^r$ over B cf. [1], [4]. $\tilde{Q}^r$ admits global crossections, [1], [4], because its standard fibre is homeomorphic with a Cartesian space of a convenable dimension. A cross-section $B \longrightarrow Q^r$ is a connection of order r on the principal bundle H and an element X of this cross-section over a point $x \in B$ will be called an element of the connection of order r. Let us consider such a cross-section Ξ and a point $x \in B$ and put $X = \Xi(x)$. We are going to define a connection form for . To begin with we have to know what is X^{-1}? X^{-1} is a non-holonomic jet $\bar{r}X$, where $\tilde{\sigma}^r$ is a non-holonomic prolongation of the mapping σ. Denote by $\tilde{\psi}^r$ a prolongation up to order r of the natural action of the groupoid Φ on the bundle H. Thus $\tilde{\psi}^r$ is an action of $\tilde{J}^r(H,\Phi)$ on $\tilde{J}^r(H,H)$. We consider some $z \in H$ and we put $\pi(z) = x$. Thus $\pi^r j_z^r 1_H \in \tilde{J}^r(H,B)$. Then we denote

$$X^{-1} \triangledown j_z^r 1_H := \tilde{\psi}^r((X^{-1})(\pi^r j_z^r 1_H),\, j_z^r 1_H)$$

The result is an element of $\tilde{J}^r(H, \pi^{-1}(x))$, [1], [4]. Then we define $T^r H$ as a dual space to $\tilde{J}^r(H,R)_0$, i.e. $\tilde{T}^r H = (\tilde{J}^r(H,R)_0)^*$. The element $X^{-1} \triangledown j_z^r 1_H$ gives rise to a unique linear mapping $(X^{-1} \triangledown j_z^r 1_H)^*$ of the vector space $\tilde{J}^r(\pi^{-1}(x),R)_0$ into $\tilde{J}^r(H,R)_0$ and a unique linear mapping

$(X^{-1} v\, j^r_z 1_H)_*$ of $\tilde{T}^r H$ into $\tilde{T}^r(\pi^{-1}(x))$. Given any $z \in H$ we define by $[z]$ an identification of the fibre through z with the group G such that e corresponds to z. Analogously we prolong $[z]$ to mapping

$$[z]^r_* : \tilde{T}^r_z \pi^{-1}(x) \longrightarrow \tilde{T}^r_e G .$$

Then the connection from ω of order r of Ξ is defined by

(1) $$\omega(v) = ([z]^r \cdot X^{-1} v\, j^r_z 1_H)_*(v)$$

for $v \in \tilde{T}^r_z H$. The basic references for this section are [1], [4].

II. BASIC CONSTRUCTIONS

From now we assume that there is given a Lie group and a transitive regular left action

$$\tau : K \times B \longrightarrow B \ / \ (g,m) \longmapsto \tau(g,m)$$

q being a positive integer we define $\tilde{\tau}^q(-,-)$ as a q-lift of τ, which acts on the manifold of non-holonomic frames H_q over B. Then we proceed by induction. Let X be a non--holonomic q-frame on B, i.e. a regular element of $\tilde{J}^q_0(\mathbb{R}^d, B)$ where $d = \dim B$. We put $X_1 = \tilde{j}^q_1 X$, where $\tilde{j}^q_1$ denotes a projection of jets of order q into jets of order 1. Thus X_1 is a frame of order 1 and there exists a regular local mapping $f : \mathbb{R}^d \longrightarrow B$ such that $j^1_{x|0} f(x) = X_1$. Then we put

$$\tilde{\tau}^1(g,X_1) := j^1_{x|0}\, \tau(g,f(x))$$

Let us assume that $\tilde{\tau}^{q-1}$ is defined. Then $X = j_0^1 \xi$, ξ being some cross-section in $\tilde{J}^{q-1}(\mathbb{R}^d, B)$. We put

$$\tilde{\tau}^q(g,X) := j^1_{x|0} \tilde{\tau}^{q-1}(g, \xi(x))$$

PROPOSITION 1. $\tilde{\tau}^q$ defines an associative left action, i.e.

$$\tilde{\tau}^q(k, \tilde{\tau}^q(l,-)) = \tilde{\tau}^q(kl,-)$$

By definition, a non-holonomic q-coframe on B is a regular q-jet whichs source is in M and its target is at 0 in $\mathbb{R}^d$. Let H_q^* be the bundle of q-coframes on B. Then K acts on H_q^* by the following manner:

If $Y \in H_q^*$, $a \in K$, we put $Y_1 = \mathcal{J}_1^q Y$, so that $Y = j_m^1 f$. Then we put

$$\overset{*}{\tau}{}^1(a,Y_1) := j^1_{\tau(a,m)} f(\tau(a,-))$$

Then we pass to higher order by a standard inductive proceeding.

PROPOSITION 2. There holds the following formula for the just described action of K on H_q^*

$$\overset{*}{\tau}{}^q(b, \overset{*}{\tau}{}^q(a,Y)) = \overset{*}{\tau}{}^q(ab,Y)$$

Thus $\overset{*}{\tau}{}^q$ is an associative right action.

Let X_0 be a fixed q-frame at some point $m_0 \in B$. We lead into considerations the following set of q-frames on B

$$W_q = \{ \tilde{\tau}^q(k,X_0) | k \in K \}$$

We define on W_q a projection π_q onto B by the following formula

$$\pi_q(\tilde{\tau}^q(k,X_0)) = \tau(k,m_0)$$

We denote by $\tilde{L}_q^d$ the structure of Lie group on a set $\tilde{J}_0^q(\mathbb{R}^d,\mathbb{R}^d)_0$ restricted to regular jets and we denote by

$$K_m := \{g \in K \mid \tau(g,m) = m\}$$

the stability group of τ.

PROPOSITION 3. For any $m \in B$ a mapping

$$\chi_{X_0} : K_m \longrightarrow \tilde{L}_q^d \;/\; k \longmapsto X_0^{-1} \cdot \tilde{\tau}^q(k,X_0)$$

is a homomorphism of Lie groups.

P r o o f. Given any q-frame X then there exists a unique q-coframe X^{-1} which may be viewed as follows: we consider $\mathcal{J}_1^q X = X_1$ which is a regular 1-jet, i.e. $X_1 = j_0^1 f$ so that $X_1^{-1} = j_{f(0)}^{-1} f^{-1}$ the (q-1)-coframes being defined we take a cross-section such that $X = j_0^1 \xi$, $\xi : \mathbb{R}^d \longrightarrow W_{q-1}$ and we put $X^{-1} := j^1_{x|f(0)}(\xi(x))^{-1}$.

In order to prove that χ_{X_0} is in fact a homeomorphism we use Proposition 1 and we have

$$\chi_{X_0}(kl) = X_0^{-1} \cdot (\tilde{\tau}^q(kl,X_0)) = X_0^{-1} \cdot (\tilde{\tau}^q(k,X_0 X_0^{-1} \tilde{\tau}^q(l,X_0)) =$$

$$= X_0^{-1} \cdot (\tilde{\tau}^q(k,X_0) X_0^{-1} \tilde{\tau}^q(l,X_0) = \chi_{X_0}(k)\chi_{X_0}(l)$$

We introduce the following notations:
$\tilde{G}_q(X)$ resp. $\tilde{G}_q(Y)$, is the image of K_m by χ_X, resp. χ_Y, X and Y being any two elements of the bundle W_q at m and at p respectively.

PROPOSITION 4. There exists an isomorphism $\widetilde{G}_q(X) \longrightarrow \widetilde{G}_p(Y)$ such that following diagram is commutative

$$\begin{array}{ccc} K_m & \longrightarrow & K_p \\ \downarrow & & \downarrow \\ \widetilde{G}_q(X) & \longrightarrow & \widetilde{G}_q(Y) \end{array}$$

P r o o f. Let $g \in K$ be any element which sends X to Y. Thus K_p and K_m are Adj_g - related. Let us define a mapping

$$\xi_g : \widetilde{G}_q(X) \longrightarrow \widetilde{G}_q(Y)$$

(2)

$$X^{-1}\tilde{\tau}^q(k,X) \longmapsto \tilde{\tau}^q(g^{-1},X^{-1})\,\tilde{\tau}^q(gkg^{-1},\tilde{\tau}^q(g,X))$$

Keeping in mind that $\tau^q(g,X) = Y$ we obtain

$$\chi_Y(gkg^{-1}) = Y^{-1}\tilde{\tau}^q(gkg^{-1},Y) =$$
$$= \tilde{\tau}^q(g^{-1},X^{-1})\,\tilde{\tau}^q(gkg^{-1},\tilde{\tau}^q(g,X)) = \xi_g(\chi_X(k))$$

Since we may view ξ_g to be mapping which sends any $\chi_X(k)$ to $\chi_Y(Adj_g k)$ then there holds

$$\xi_g = \chi_X = \chi_Y \circ Adj_g$$

Evidently ξ_g is an isomorphism.

The above results imply the following

THEOREM 5. Given any fixed frame $X_o \in H_q$ then there exists a unique frame bundle W_q over B with the structure group $\widetilde{G}_q$, the image by χ_{X_o} of the isotropy group K_{m_o}.
π_q is the projection.

III. ELEMENTS OF INVARIANT CONNECTIONS

Let us fix any point $m \in B$. Denote by $\mathbb{K}$ and respectively, by $\mathbb{K}_m$ the Lie algebras of K and of K_m. Let D_m be any complementary space with respect to $\mathbb{K}_m$ in $\mathbb{K}$. We choose a linear basis $[e_1,\dots,e_d]$ in D_m. In some neighbourhood U of 0 in $\mathbb{R}^d$ there is defined a mapping

$$[t^1,\dots,t^d] \longmapsto \exp\Big(\sum_{\alpha=1}^{d} t^\alpha e_\alpha\Big) =: g(t)$$

Let us consider the mapping

$$t \longmapsto \tau(g(t),m) \tag{3}$$

This mapping is a diffeomorphism of U to some neighbourhood V of m.

Let

$$w : V \longrightarrow \mathbb{R}^d$$

be reciprocal to the mapping (3). We have $w(m) = 0$.

X being a frame in the fibre $\pi_q^{-1}(m)$ we consider the mapping θ defined by

$$\theta(t,m,X) := \tilde{\tau}^q(g(t),X)$$

Thus $\theta(t,m,-)$ maps the fibre $\pi_q^{-1}(m)$ to the fibre $\pi^{-1}(\tau(g(t),m)$. We remark that if $t = w(p)$ for some $p \in V$, then we have

$$\theta(t,m,-) : \pi_q^{-1}(m) \longrightarrow \pi_q^{-1}(p)$$

THEOREM 6. $\Theta(t,m,-)$ <u>is a fibre morphism of</u> $\pi_q^{-1}(m)$ <u>to</u> $\pi_q^{-1}(\tau(g(t),m))$.

P r o o f. We have to show that $\Theta(t,m,-)$ commutes with the cononical action of $\tilde{G}_q$, that means, the following diagram is commutative:

$$\begin{array}{ccc} \pi_q^{-1}(m) & \xrightarrow{\Theta(t,m,-)} & \pi_q^{-1}(p) \\ \chi_z(h)\downarrow & & \downarrow \chi_w(Adj_{g(t)}h) \\ \pi_q^{-1}(m) & \xrightarrow{\Theta(t,m,-)} & \pi_q^{-1}(p) \end{array}$$

for any K_m and by any choice of $Z \in \pi_q^{-1}(m)$. We see that $Adj_{g(t)}h \in K_p$ and $W \in \pi_q^{-1}(p)$ is a map of Z by $\tilde{\tau}^q(g(t),-)$. Then the group $\tilde{G}_q$ acts on the fibre $\pi_q^{-1}(m)$ by the following rule

$$T : \tilde{G}_q \times \pi_q^{-1}(m) \longrightarrow \pi_q^{-1}(m)$$

$$(\chi_Z(h),X) \longmapsto \tilde{\tau}^q(kh, \tilde{\tau}^q(k^{-1},X))$$

Here $k \in K$ is such that $X = \tilde{\tau}^q(k,Z)$. Thus T defines a right action. Consider the mapping $\pi_q^{-1}(m) \longrightarrow \pi_q^{-1}(p)$ given by

$$X \longmapsto \Theta(t,m,T(\chi_Z(h),X))$$

we have

$$\Theta(t,m, \tilde{\tau}^q(kh,Z)) = \tilde{\tau}^q(g(t), \tilde{\tau}^q(kh,Z)) = \tilde{\tau}^q(g(t)kh,Z) \tag{4}$$

On the other hand we have

$$\Theta(t,m,X) = \Theta(t,m, \tilde{\tau}^q(k,Z)) = \tilde{\tau}^q(g(t), \tilde{\tau}^q(k,Z))$$

In view of formula (2) we have

$$\chi_W(h) = \overset{\#}{\tau}{}^q((g(t))^{-1}, Z^{-1})\,\tilde{\tau}^q(g(t)h(g(t))^{-1}, \tilde{\tau}^q(g(t),Z)))$$

By consequence

$$T(\bar{\chi}_W(h), \Theta(t,m,X)) =$$
$$= \tilde{\tau}^q(g(t), \tilde{\tau}^q(k,Z))\overset{\#}{\tau}{}^q(g(t),Z^{-1})\cdot\tilde{\tau}^q(g(t)h(g(t))^{-1}, \tilde{\tau}^q(g(t),Z)) =$$
$$= \tilde{\tau}^q(g(t)kh(g(t))^{-1}, \tilde{\tau}^q(g(t),Z)) = \tilde{\tau}^q(g(t)kh, Z)$$

If we compare this result with (4) then we finish the proof.

Let us denote by Φ_q the groupoid associated with W_q. Thus each $\Theta(t,m,-)$ is an element of Φ_q. Then we define an action of the group K on these elements of Φ_q. We put

$$k * \Theta(t,m,-) := \Theta(t, \bar{\tau}(k,m),-)$$

If $X \in \pi_q^{-1}(\tau(k,m))$ then we have $k * \Theta(t,m,X) =$
$= \Theta(t, \tau(k,m), X) = \tilde{\tau}^q(g(t),X) \in \pi_q^{-1}(\tau(g(t)k,m))$.

PROPOSITION 7. If $\tau(k,m) = \tau(1,m)$ then we have

$$k * \Theta(t,m,-) = 1 * \Theta(t,m,-)$$

IV. BUNDLES OF INVARIANT ELEMENTS OF THE CONNECTION

Let us define a cross-section

$$C : B \longrightarrow \tilde{J}^r(B, \Phi_q)$$

by $C_p := j^r_{s|p} k * \Theta(w(\tau(k^{-1},s)),m,-)$ where k is such that

$\tau(k,m) = p$. In view of Proposition 7, C_p does not depend on the choice of k. Then we see that C is a cross-section in the bundle of elements of the connection, that means:

1° $\alpha(C_p) = p$

2° $\beta(C_p) = \tilde{p}$

3° $a^r(C_p) = \rho^r_x$

4° $b^r(C_p) = j^r_p 1_B$

Remark that

$$C_m = j^r_{s|m}\,\theta(w(s),m,-)$$

Then we put by definition

$$\tilde{\tau}^r C_m := j^r_{t|m} k * \theta(w(t),m,-)$$

and

$$C_m \tilde{\tau}^r_{k^{-1}} := j^r_{t|\tau_k(m)}\,\theta(w(\tau(k^{-1},t)),m,-)$$

The following identities follow easily by definitions

$$C_{\tau(k,m)} = \tilde{\tau}^r_k C_m \tilde{\tau}^r_{k^{-1}}$$

and

$$C^{-1}_{\tau(k,m)} = \tilde{\tau}^r_k C^{-1}_m \tilde{\tau}^r_{k^{-1}} \tag{5}$$

Let us turn to the constructions in the preceeding chapter. The construction of the mapping w does depend on a choice of the complementary space D_m but it does not depend on a choice of the linear basis in D_m. Thus C_m and, by consequence, the cross-section $p \longmapsto C_p$ depends only on the choice of D_m. We have seen that each C_p is the element of the connection in the sense indicated in our preliminaries. Let us recollect the notations.

If $Z \in W_q$ then $[Z]$ is a diffeomorphism of the fibre through Z to the group $\tilde{G}_q$ and $[Z](Z) = e$ (neutral element in the group $\tilde{G}_q$). Then we prolong $[Z]$ to a mapping $[Z]^r_*$ of $\tilde{T}^r_Z W_q = (\tilde{J}^r_Z(W_q,R)_o)^*$ onto $\tilde{T}^r_e \tilde{G}_q$. Then we have to prolong $\psi : \Phi \times W_q \longrightarrow W_q$ to $\tilde{\psi}^r$ which acts on $\tilde{J}^r(W_q, \Phi_q) \times \tilde{J}^r(W_q,W_q)$ and maps it to $\tilde{J}^r(W_q,W_q)$. Then the value of the form of our connection ω^r_q on the element $y \in \tilde{T}^r W_q$ at the point Z is, by definition

$$(6) \quad \omega^r_q(y) = \left\{[Z]^r \cdot \tilde{\psi}^r(C^{-1}_m j^r_Z \pi_q, j^r_Z 1_{W_q})\right\}_*(y), \qquad \pi_q(Z) = m$$

The compositions inside the parantheses are to be understood as a non-holonomic jet composition. The group K acts on $\tilde{T}^r W_q$ by means of a non-holonomic lifting of τ. This lift will be denoted by $\tilde{\tau}^r$.

A connection is invariant under the action of K iff its form satisfies

$$\omega^r_q(\tilde{\tau}^r(k,y)) = \omega^r_q(y)$$

for each $k \in K$ and each $y \in \tilde{T}^r W_q$.

THEOREM 8. <u>The connection defined above by</u> C <u>is invariant under</u> K.

P r o o f. In view of (6) we have

$$\omega^r_q(\tilde{\tau}^r(k,y)) =$$

$$= \left\{[\tilde{\tau}^q(k,Z)]^r \tilde{\psi}^r(C^{-1}_{\tau(k,m)} j^r_{\tilde{\tau}^q(k,Z)} \pi_q, j^r_{\tilde{\tau}^q(k,Z)} 1_{W_q})\right\}_*(\tilde{\tau}^r(k,y))$$

We make use of (5). Thus we have

(7) $$\omega_q^r(\tilde{\tau}^r(k,y)) =$$

$$= \{[\tilde{\tau}^q(k,Z)]^r \tilde{\psi}^r(\tilde{\tau}_k^r C_m^{-1} \tilde{\tau}^r_{k^{-1}} j^r_{\tilde{\tau}^q(k,Z)} \pi_q, j^r_{\tilde{\tau}^q(k,Z)} 1_{W_q})\}_*(\tilde{\tau}^r(k,y))$$

First we notice that

$$\tilde{\tau}^r_{k^{-1}} j^r_{\tilde{\tau}^q(k,Z)} \pi_q = (j^r_Z \pi_q)\tilde{\tau}^r_{k^{-1}}$$

and

$$j^r_{\tilde{\tau}^q(k,Z)} 1_{W_q} = (j^r_Z 1_{W_q})\tilde{\tau}^r_{k^{-1}}$$

Consider the mapping $([Z]^r \tilde{\tau}^r_{k^{-1}})_*$ which is a linear mapping from $\tilde{T}^r_{\tilde{\tau}^q(k,Z)} W_q$ into $\tilde{T}^r_e \tilde{G}_q$. We have evident equality

$$([Z]^r \tilde{\tau}^r_{k^{-1}})_* = [\tilde{\tau}(k,Z)]^r_*$$

We substitute these above equalities to (7) and we obtain

$$\omega^r(\tilde{\tau}^r(k,y)) =$$

$$= \{[Z]^r \tilde{\tau}^r_{k^{-1}} \tilde{\psi}^r(\tilde{\tau}^r_k C_m^{-1}(j^r_Z \pi_q)\tilde{\tau}^r_{k^{-1}}, (j^r_Z 1_{W_q})\tilde{\tau}^r_{k^{-1}}\}_*(\tilde{\tau}^r(k,y)) =$$

$$= \{[Z]^r \tilde{\tau}^r_{k^{-1}} \tilde{\tau}^r_k \tilde{\psi}(C_m^{-1} j^r_Z \pi_q, j^r_Z 1_{W_q})\}_*((\tilde{\tau}^r_{k^{-1}})_*(\tilde{\tau}^r_k)_*(y)) =$$

$$= \{[Z]^r \tilde{\psi}^r(C_m^{-1} j^r_Z \pi_q, j^r_Z 1_{W_q})\}_*(y) = \omega_q^r(y)$$

$\tilde{\tau}^r_k$ is $\tilde{\tau}^r(k,-)$ for abbreviation of notations.

The first of authors of this paper has proved in [7] the following theorem.

THEOREM 9. <u>If</u> $q = r = 1$ <u>then above invariant connection</u> ω_1^1 <u>is flat.</u>

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STRESZCZENIE

Rozważamy rozmaitość B o wymiarze d, na której działa lewostronnie grupa Liego K. Działanie to przedłużamy (na ogół nie holonomicznie) do działania grupy K na rozmaitości żetów $\tilde{J}^q_0(R^d,B)$. Z przedłużeniem tym wiąże się konstrukcja pewnej wiązki reperów q-tego rzędu nad B, niezmienniczej względem K. W tej wiązce konstruujemy niezmienniczą koneksję r-tego rzędu oraz formę tej koneksji.

Резюме

В данной работе рассматривается многообразие B размерности d,на котором действует с лева группа Ли K. Это действие продолжаем неголономически к действию группы на многообразии струи $\tilde{J}^q_0(R^d,B)$инвариантного относительно к действию группы K. Строится инвариантная связность порядка q и форма этой связности в раслоённом пространстве реперов порядка q над B, являющемся определенной редукцией пучка всех q-реперов. Построено также форму такой связности. Дальнейшие ее свойства изучаются в последующей работе [8]