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Expected Mean Squares and Tests of Significance for Mixed Model 3×3 with Interaction in the Case of Non-Orthogonal Data

Wartości oczekiwane średnich kwadratów i testy istotności dla mieszanego modelu 3×3 z interakcją w przypadku danych nieortogonalnych

Математическое ожидание средних квадратов и критерий значимости для смешанной модели 3×3 с взаимодействием в случае неортогональных данных

1. Introduction

In the preceding paper [1] we did not give the explicit form of the expectation of mean square for interaction AB for the mixed general model $I \times J$ (any I and any $J > 2$) in the case of non-orthogonal data.

The aim of the present paper is to find the expectation mentioned above in the case of mixed model 3×3 under the same assumptions (cf. [1], sec. 4) as before. Moreover, we wish to present the tests for testing the hypotheses concerning the main effects A, B and interaction effects.

2. Notation

1. $y_{ijl} = \underbrace{\mu + a_i}_{\text{fixed}} + \underbrace{b_j + c_{ij} + e_{ijl}}_{\text{random}}; \; l = 1, 2, \ldots, n_{ij};$

$n_{ij} \geqslant 1$ for all $i, j = 1, 2, 3$.

All the symbols except for these given under items 24 and 25 (cf. [1]) are valid in the present paper. We shall also use the following ones:

2. $h_{jj} = n_{.j} - \sum_{i=1}^{3} \frac{n_{ij}^2}{n_{i.}}; \quad j = 1, 2, 3;$

$$h_{jj'} = h_{j'j} = -\sum_{i=1}^{3} \frac{n_{ij} n_{ij'}}{n_{i.}} \quad \text{for} \quad j \neq j'; \quad j, j' = 1, 2, 3.$$

3. $p_{11} = h_{22}+h_{33}-2h_{23}, \quad p_{22} = h_{11}+h_{33}-2h_{13}, \quad p_{33} = h_{11}+h_{22}-2h_{12},$

$$p_{12} = p_{21} = -h_{12}+h_{13}+h_{23}-h_{33},$$

$$p_{13} = p_{31} = h_{12}-h_{13}-h_{22}+h_{23},$$

$$p_{23} = p_{32} = -h_{11}+h_{12}+h_{13}-h_{23}.$$

4. $D = p_{11}p_{22}-p_{12}^2$

5. The orthogonal case $n_{ij} = k =$ constant:

$$h_{11} = h_{22} = h_{33} = 2k; \quad h_{12} = h_{13} = h_{23} = -k;$$

$$p_{11} = p_{22} = p_{33} = 6k; \quad p_{12} = p_{13} = p_{23} = -3k; \quad D = 27k^2.$$

6. $M_j = 2n_{1j}\left\{n_{.j}-n_{.j}\frac{n_{1j}}{n_{1.}} - \sum_i \frac{n_{ij}^2}{n_{i.}} + \frac{1}{n_{1.}}\sum_i\left[\frac{n_{ij}}{n_{i.}}\left(\sum_j n_{1j}n_{ij}\right)\right]\right\}.$

7. $u_{jk} = -\frac{n_{1k}n_{1j}}{n_{1.}}(n_{.j}+n_{.k})+h_{jk}(n_{1k}+n_{1j})+\frac{n_{1k}}{n_{1.}}\sum_l^3 \frac{n_{lj}\sum_j n_{1j}n_{lj}}{n_{l.}}+$

$$+\frac{n_{1j}}{n_{1.}}\sum_l^3 \frac{n_{lk}\left(\sum_j n_{1j}n_{lj}\right)}{n_{l.}}, \quad j \neq k; \quad j,k = 1,2,3.$$

8. $f_i = n_{i.}-\frac{\sum_j n_{ij}^2}{n_{i.}} - \frac{1}{D}\left[\sum_{j=1}^3 p_{jj}n_{ij}^2\left(1-\frac{2n_{ij}}{n_{i.}}+\frac{\sum_j n_{ij}^2}{n_{i.}}\right)+\right.$

$$\left.+2\sum_{\substack{j<j'\\ j,j'=1,2,3}} p_{jj'}\frac{n_{ij}n_{ij'}}{n_{i.}}\left(\frac{\sum_j n_{ij}^2}{n_{i.}}-n_{ij}-n_{ij'}\right)\right] \quad \text{for} \quad i = 1,2,3.$$

9. $g_{ii'} = \frac{2}{D}\left[\sum_j p_{jj}n_{ij}n_{i'j}\left(1-\frac{n_{ij}}{n_{i.}}-\frac{n_{i'j}}{n_{i'.}}+\frac{\sum_j n_{ij}n_{i'j}}{n_{i.}n_{i'.}}\right)+\right.$

$$+\sum_{\substack{j<j'\\ j,j'=1,2,3}} p_{jj'}\left(\frac{n_{ij}n_{i'j'}+n_{ij'}n_{i'j}}{n_{i.}n_{i'.}}\sum_j n_{ij}n_{i'j}-\frac{n_{ij}n_{ij'}(n_{i'j}+n_{i'j'})}{n_{i.}}-\right.$$

$$\left.\left.-\frac{n_{i'j}n_{i'j'}(n_{ij}+n_{ij'})}{n_{i'.}}\right)\right] \quad \text{for} \quad i,i' = 1,2,3.$$

3. Assumptions

Assumptions for the mixed model 3×3 are the same as in the general case $I\times J$ (cf. [1]). To obtain them, it is sufficient to consider $I = J = 3$. Analogously, the definitions of main effects and interaction effects are unchanged; we accept $v_i = w_j = 1/3$; $i, j = 1, 2, 3$. Then

$$W_i = 9\left(\sum_j \frac{1}{n_{ij}}\right)^{-1}, \quad V_j = 9\left(\sum_i \frac{1}{n_{ij}}\right)^{-1}.$$

It means that we are interested in the method of weighted squares of means and that subclass numbers in the classes of the population are identical. Thus the restrictions are as follows:

$$\sum_i^3 a_i = \sum_i^3 c_i(v) = 0 \quad \text{for} \quad \text{all } v, \quad E\big(b(v)\big) = E\big(c_i(v)\big) = 0 \quad \text{for all } i.$$

Consequently, the relations between $\mathrm{Var}\big(b(v)\big)$, $\mathrm{Cov}\big(b(v), c_i(v)\big)$, $\mathrm{Cov}\big(c_i(v), c_{i'}(v)\big)$, $\mathrm{Var}\big(c_i(v)\big)$ and $\sigma_{ii'}$ hold as in sec. 7 of [1]; $i, i' = 1, 2, 3$.

4. Expected mean squares

Theorem. *Expected mean square for interaction for mixed model* 3×3

$$y_{ijl} = \underbrace{\mu + a_i}_{\text{fixed}} + \underbrace{b_j + c_{ij} + e_{ijl}}_{\text{random}}$$

$$l = 1, 2, \ldots, n_{ij}; \quad n_{ij} \geqslant 1 \text{ for all } i, j = 1, 2, 3$$

under assumptions presented above (cf. sec. 3) *is of the following form*:

$$(2) \qquad E(MS_{AB}) = \sigma_e^2 + \frac{1}{4}\sum_{i=1}^{3} f_i \mathrm{Var}\big(c_i(v)\big) - \frac{1}{4}\sum_{\substack{i<i'\\ i,i'=1,2,3}} g_{ii'}\mathrm{Cov}\big(c_i(v), c_{i'}(v)\big)$$

where f_i *and* $g_{ii'}$ *are given in sec.* 2.

Proof: From Table 1 (cf. [1]) we have for $I = J = 3$:

$$(3) \qquad E(MS_{AB}) = \frac{3}{2}\sigma_e^2 + \frac{1}{4}\left\{\left(n - \frac{\sum_j n_{\cdot j}^2}{n_{i.}}\right)\mathrm{Var}\big(b(v)\big) + \right.$$

$$+ \sum_i\left(n_{i.} - \frac{\sum_j n_{ij}^2}{n_{i.}}\right)\mathrm{Var}\big(c_i(v)\big) +$$

$$\left. + 2\sum_i \mathrm{Cov}\big(b(v), c_i(v)\big)\left(n_{i.} - \frac{\sum_j n_{ij}^2}{n_{i.}}\right) - E\sum_j \hat\beta_j Q_{.j}\right\}.$$

Thus in order to obtain $E(MS_{AB})$ explicitly it is sufficient to calculate $E(\sum_j \hat{\beta}_j Q_{.j})$, where the $\hat{\beta}_j$'s are estimates under the following model without interaction:

(4) $$y_{ijl} = \mu + a_i + \beta_j + e_{ijl}.$$

Let us calculate $E\sum_j \beta_j Q_{.j}$ in the case when $J = 3$. From the normal equations

(5) $$h_{j1}\hat{\beta}_1 + h_{j2}\hat{\beta}_2 + h_{j3}\hat{\beta}_3 = Q_{.j}; \quad j = 1, 2, 3$$

under the unweighted restriction

(6) $$\hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3 = 0$$

we obtain

(7) $$\hat{\beta}_i = \frac{1}{D}\sum_j p_{ij}Q_{.j}; \quad i = 1, 2, 3.$$

Hence, we find

(8) $$E\sum_j \hat{\beta}_j Q_{.j} = \frac{1}{D}\Big(\sum_j p_{jj}EQ_{.j}^2 + 2\sum_{\substack{j<j'\\ j,j'=1,2,3}} p_{jj'}EQ_{.j}Q_{.j'}\Big).$$

The expectations: $E(Q_{.j}^2)$; $j = 1, 2, 3$; and $E(Q_{.j}Q_{.j'})$; $j \neq j'$; $j, j' = 1, 2, 3$; were given in sections 9 and 10 of [1] for any I and any J. Now, we are going to use these expressions.

Let us calculate separately the following five coefficients contained in $E(MS_{AB})$:

a) coefficient with σ_e^2,

b) ——,,—— with $\mathrm{Var}\big(b(v)\big)$,

c) ——,,—— with $\mathrm{Cov}\big(b(v), c_i(v)\big)$,

d) ——,,—— with $\mathrm{Cov}\big(c_i(v), c_{i'}(v)\big)$, $i \neq i'$,

e) ——,,—— with $\mathrm{Var}\big(c_i(v)\big)$.

Ad a). It is easy to find that the coefficient with σ_e^2 in $E\sum_{j=1}^{3}\hat{\beta}_j Q_{.j}$ is equal to 2, i. e. that the coefficient with σ_e^2 in $E(MS_{AB})$ is equal to 1.

Ad b). Consider that the coefficients with $\mathrm{Var}\big(b(v)\big)$ in $E(Q_{.j}^2)$ and in $E(Q_{.j}Q_{.k})$ are equal to

$$n_{.j}^2 - 2n_{.j}\sum_{i=1}^{3}\frac{n_{ij}^2}{n_{i.}} + \sum_{l=1}^{3}\Big(\sum_{i=1}^{3}\frac{n_{ij}n_{il}}{n_{i.}}\Big)^2$$

and to

$$\sum_{l=1}^{3}\Big(\sum_{i=1}^{3}\frac{n_{ij}n_{il}}{n_{i.}}\Big)\Big(\sum_{i=1}^{3}\frac{n_{ik}n_{il}}{n_{i.}}\Big)+(n_{.j}+n_{.k})h_{jk},$$

$j\neq k;\ j,k=1,2,3$ respectively. Using the identity

$$h_{11}h_{22}h_{33}-h_{11}h_{23}^2-h_{22}h_{13}^2-h_{33}h_{12}^2+2h_{12}h_{13}h_{23}\equiv 0$$

we can prove that the coefficient with $\operatorname{Var}(b(v))$ in $E(SS_{AB})$ is equal to zero. It means also that the coefficient with $\operatorname{Var}(b(v))$ in $E\big(\sum_j\hat{\beta}_jQ_{.j}\big)$ is equal to (cf. Table 1):

$$n-\sum_i\frac{\sum_j n_{ij}^2}{n_{i.}}=h_{11}+h_{22}+h_{33}. \tag{9}$$

Ad c). The calculations connected with this step are long and tedious.

We want to prove that the coefficient with $\operatorname{Cov}[b(v),c_i(v)]$ is not included in $E(MS_{AB})$. In order to show this it is sufficient to prove that

$$2\sum_i^{3}\Big(n_{i.}-\frac{\sum_j^{3} n_{ij}^2}{n_{i.}}\Big)\operatorname{Cov}\big(b(v),c_i(\mathrm{v})\big) \tag{10}$$

is equal to the expression with $\operatorname{Cov}\big(b(v),c_i(v)\big)$ in $E\big(\sum_j\hat{\beta}_jQ_{.j}\big)$ (cf. Table 1, [1]). Because of the symmetry it is sufficient to find that the coefficient with $\operatorname{Cov}\big(b(v),c_1(v)\big)$ is the same as in $G=E\big(\sum_j\hat{\beta}_jQ_{.j}\big)$ and in $H=X-$ $-E(SS_{AB})$ where $E(SS_{AB})=X-E\big(\sum_j^{3}\hat{\beta}_jQ_{.j}\big)$ i. e. that

$$\frac{1}{D}(p_{11}M_1+p_{22}M_2+p_{33}M_3+2p_{12}u_{12}+2p_{13}u_{13}+2p_{23}u_{23})= \tag{11}$$
$$=2\big(n_{1.}-\sum_j n_{1j}^2/n_{1.}\big).$$

Let us do this. After multiplying both sides of (11) by D and after expressing D, M, p, and u as the functions of n_{ij}, $n_{i.}$, $n_{.j}$, we reduce $n_{1.}D$. Then we find that all the fractions with denominators $n_{1.}$, $n_{1.}^3$, $n_{1.}^2$, $n_{1.}^2n_{2.}$, $n_{1.}^2n_{3.}$, $n_{1.}n_{2.}^2$, $n_{1.}n_{3.}^2$, $n_{2.}^2$, $n_{3.}^2$, $n_{2.}^3$, $n_{2.}^2n_{3.}$, $n_{3.}^2\,n_{2.}$, and $n_{3.}^3$ in G and H are identical. From the remaining expressions we form the fractions with the same denominators $n_{1.}\,n_{2.}\,n_{3.}$ and we verify directly that theirs nominators in H and in G are identical. The proof in this step is concluded.

Ad d) and e). Using the results of sections 8, 9 and 10 (cf. [1]) and the formula 8 we prove directly that the coefficient with $\operatorname{Var}(c_i(v))$ in

Table 1

Mixed model 3×3 with unequal subclass numbers (significant interaction) $n_{ij} \geqslant 1$ for all i, j

$$y_{ijl} = \mu + a_i + b_j + c_{ij} + e_{ijl}; \; i = 1, 2, 3; \; j = 1, 2, 3; \; l = 1, 2, \ldots, n_{ij}$$

The unweighted restrictions: $\sum^{I} a_i = \sum^{I} c_{ij} = 0$ for all j; $E(b(v)) = E(c_i(v)) = 0$ for all i (method of weighted squares of means)

Source of variation	D. F	Sums of squares	$E(MS)$ under mixed model
1.A (fixed)	2	$SS_A = \sum_{i=1}^{3} W_i(\hat{A}_i - \bar{A})^2 = \sum_{i=1}^{I} W_i \hat{A}_i^2 - \frac{(\sum_i W_i \hat{A}_i)^2}{\sum_i W_i}$	$E(MS_A) = \sigma_e^2 + \frac{1}{2}\Big\{\sum_{i=1}^{I} W_i \Big(a_i - \frac{\sum W_i a_i}{\sum W_i}\Big)^2 + \frac{1}{3}\Big[\sum_{i=1}^{I} W_i \operatorname{Var}(c_i(v)) - \frac{\sum W_i^2 \operatorname{Var}(c_i(v))}{\sum W_i}\Big] - \frac{2}{\sum W_i} \sum_{\substack{i<i' \\ i,i'=1,2,3}} W_i W_{i'} \operatorname{Cov}(c_i(v), c_{i'}(v))\Big\}$ When $n_{ij} = k = \text{const}$, we obtain $E(MS_A) = \sigma_e^2 + k \sum_{i=1}^{3} a_i^2 + \frac{k}{2} \sum_{i=1}^{3} \operatorname{Var}(c_i(v))$ as it should be (cf. [3])

2. B (random)	2	$SS_B = \sum_{j=1}^{3} V_j(\hat{B}_j - \overline{B})^2 = \sum_{j=1}^{J} V_j \hat{B}_j^2 - \frac{(\sum V_j \hat{B}_j)^2}{\sum_j V_j}$	$E(MS_B) = \sigma_e^2 + \frac{1}{2}\left[\sum_{j=1}^{J} V_j - \frac{\sum V_j^2}{\sum V_j}\right] \mathrm{Var}(b(v))$ When $n_{ij} = k =$ const. we obtain $E(MS_B) = \sigma_e^2 + 3k\,\mathrm{Var}(b(v))$ as it should be (cf. ibid.)
3. AB (random) interaction	4	(least-squares analysis) $SS_{AB} = \sum_i^3 \sum_j^3 n_{ij}(\bar{y}_{ij.} - \bar{y}_{.j.})^2 - \sum_i \hat{a}_i Q_{i.} = \sum_i^3 \sum_j^3 n_{ij}(\bar{y}_{ij.} - \bar{y}_{i..})^2 - \sum_l \beta_j Q_{.j}$	$E(MS_{AB}) = \frac{3}{2}\sigma_e^2 + \frac{1}{4}\left\{\left(n - \sum_i \frac{\sum n_{ij}^2}{n_{i.}}\right)\mathrm{Var}(b(v)) + \sum_{i=1}^{3}\left(n_{i.} - \frac{\sum n_{ij}^2}{n_{i.}}\right)\mathrm{Var}\,c_i(v) + 2\sum_{i=1}^{3}\mathrm{Cov}(b(v), c_i(v)) \times \left(n_{i.} - \frac{\sum_j^3 n_{ij}^2}{n_{i.}}\right) - E\sum^{J}\hat{\beta}_j Q_{.j}\right\} = \sigma_e^2 + \frac{1}{4}\sum_{i=1}^{3} f_i \mathrm{Var}(c(v)) - \frac{1}{4}\sum_{\substack{i<i' \\ i,i'=1,2,3}} g_{ii'}\mathrm{Cov}(c_i(v), c_{i'}(v))$ When $n_{ij} = k =$ constant we obtain $E(MS_{AB}) = \sigma_e^2 + \frac{k}{2}\sum_{i=1}^{3}\mathrm{Var}(c_i(v))$
4. Error (within subclasses)	$n-9$	$SS_e = \sum_i \sum_j \sum_l (y_{ijl} - \bar{y}_{ij.})^2$	$E(MS_e) = \sigma_e^2$

$E(MS_{AB})$ is equal to $1/4\ f_i$, and that the coefficient with $\mathrm{Cov}(c_i(v), c_{i'}(v))$ is equal to $-1/4\ g_{ii'}$, $i < i'$; $i, i' = 1, 2, 3$.

The proof of the theorem is concluded. Thus $E(MS_{AB})$ depends neither on $\mathrm{Var}(b(v))$ nor on $\mathrm{Cov}(b(v), c_i(v))$.

Remark 1. From the proof presented above it follows that

$$(12)\qquad E\sum_j^3 \hat{\beta}_j Q_{.j} = 2\sigma_e^2 + \left(n - \sum_i \frac{\sum_j n_{ij}^2}{n_{i.}}\right)\mathrm{Var}(b(v)) + 2\sum_i \left(n_{i.} - \frac{\sum_j n_{ij}^2}{n_{i.}}\right)\times$$

$$\times \mathrm{Cov}(b(v), c_i(v)) + \sum_i^3 \left(n_{i.} - \frac{\sum_j n_{ij}^2}{n_{i.}} - f_i\right)\mathrm{Var}(c_i(v)) +$$

$$+ \sum_{\substack{i<i' \\ i,i'=1,2,3}} g_{ii'}\mathrm{Cov}(c_i(v), c_{i'}(v)).$$

Remark 2. From the first expression for $E(MS_{AB})$ of Table 1 (cf. [1]) and from (2) we find

$$(13)\qquad E\Sigma\hat{a}_i Q_{i.} = 2\sigma_e^2 + \sum_i \sum_j n_{ij}\left(a_i - \frac{\sum_i n_{ij} a_i}{n_{.j}}\right)^2 + \sum_i (\eta_i - f_i)\mathrm{Var}(c_i(v)) -$$

$$-2\sum_{\substack{i<i' \\ i,i'=1,2,3}} \left[\left(u_{ii'} - \frac{1}{2} g_{ii'}\right)\mathrm{Cov}(c_i(v), c_{i'}(v))\right].$$

Remark 3. In the orthogonal case: $n_{ij} = k =$ constant we obtain from (2):

$$(14)\qquad E(MS_{AB}) = \sigma_e^2 + \frac{k}{2}\sum_{i=1}^3 \mathrm{Var}(c_i(v)),$$

as it is should be (cf. [2]).

In fact, because of the restriction $\sum_{i=1}^3 c_{ij} = 0$ the $Q_{.j}$ and consequently $E\Sigma\hat{\beta}_j Q_{.j}$ do not depend on c_{ij}. We find

$$(15)\qquad f_i = n_{i.} - \frac{\sum_j n_{ij}^2}{n_{i.}} = 3k - \frac{3k^2}{3k} = 2k$$

and

$$(16)\qquad g_{ii'} = 0 \quad \text{for all} \quad i, i' = 1, 2, 3.$$

The expected mean squares for the main effects A and B and for error are obtained directly from Table 1 (cf. [1]). The corresponding values are given in the Table on pp. 90-91.

5. Tests

1. When the hypothesis H_A : all $a_i = 0$ is true we can verify for the mixed model 3×3 that $E(MS_A) \neq E(MS_{AB})$, but both these expressions depend (cf. Table 1) on the same components, i. e. on σ_e^2, $\mathrm{Var}\big(c_i(v)\big)$ and on $\mathrm{Cov}\big(c_i(v), c_{i'}(v)\big)$; however, they depend neither on $\mathrm{Var}\big(b(v)\big)$ nor on $\mathrm{Var}\big(b(v), c_i(v)\big)$. Hence, the approximate test F for testing the significance of the A effects is based on

$$F_A = MS_A/G \tag{17}$$

where G is a linear combination of MS_e, MS_A and MS_{AB}. The corresponding degrees of freedom are given by the Satterthwaite's method [2].

It deserves attention that in the case of mixed model $I\times 2$ we have the equality $E(MS_A) = E(MS_{AB})$ when the hypothesis that all $a_i = 0$; $i = 1, 2, \ldots, I$; is true. As we already known [1] this equality does not hold for the model 3×3.

2. For testing the hypothesis

$$H_B\colon \mathrm{Var}\big(b(v)\big) = 0$$

we can use the ratio $F_B = MS_B/MS_e$.

3. The hypothesis H_{AB} that $\mathrm{Cov}\big(c_i(v), c_{i'}(v)\big) = 0$ for all $i, i' = 1, 2, 3$ can be verified by using the ratio $F_{AB} = MS_{AB}/MS_e$.

Remark. The results obtained in sec. 4 suggest that when the hypothesis H_A: all $a_i = 0$ is true then $E(MS_A) \neq E(MS_{AB})$ for the mixed model $I\times J$ $(J > 2)$. It seems that both expressions $E(MS_A)$ and $E(MS_{AB})$ depend on σ_e^2, $\mathrm{Var}\big(c_i(v)\big)$ and on $\mathrm{Cov}\big(c_i(v), c_{i'}(v)\big)$ but they depend neither on $\mathrm{Var}\big(b(v)\big)$ nor on $\mathrm{Cov}\big(b(v), c_i(v)\big)$. It remains to be proved.

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Streszczenie

Przy założeniach podanych w paragrafie 4 pracy [1] znaleziono wartości oczekiwane dla głównych efektów, interakcji i dla błędu modelu mieszanego 3×3 w przypadku danych nieortogonalnych. Podano przybliżone testy istotności F dla zweryfikowania hipotez: 1^o $H_A: a_i = 0$, że wszystkie stałe efekty są równe zeru, 2^o H_B: $\mathrm{Var}(b(v)) = 0$, że wariancja efektu losowego B jest zerem i 3^o $H_{AB}: \mathrm{Cov}(c_i(v)), c_{i'}(v)) = 0$ dla $i, i' = 1, 2, 3$ że kowariancje między efektami interakcyjnymi są równe zeru. W przypadku modelu mieszanego 3×3 i prawdziwości hipotezy H_A mamy $E(MS_A) \neq E(MS_{AB})$ podczas gdy przy tychże założeniach dla modelu mieszanego $I\times 2$ zachodzi równość $E(MS_A) = E(MS_{AB})$.

Резюме

При предположениях данных в параграфе 4 работы [1] найдено математические ожидания для главных эффектов, эффектов взаимодействия и для ошибки смешанной модели 3×3 в случае неортогональных данных. Дано приближенные критерии значимости F для проверки гипотез: 1° $H_A: a_i = 0$, что все постоянные эффекты равны нулю, 2° $H_B: \mathrm{Var}(b(v)) = 0$, что дисперсия случайного эффекта равна нулю, 3° $H_{AB}: \mathrm{Cov}(c_i(v), c_{i'}(v)) = 0$ для $i, i' = 1, 2, 3$, что ковариация между эффектами и взаимодействиями равны нулю. В случае смешанной модели 3×3 и справедливости гипотезы H_A имеем $E(MS_A) \neq E(MS_{AB})$ в то время когда при тех-же предположениях для смешанной модели $I\times 2$ имеет место $E(MS_A) = E(MS_{AB})$.